A model for influence of exercise on formation and growth of tissue bubbles during altitude decompression

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1Department of Medicine, Baylor College of Medicine, Houston 77030; 2Medical Sciences Division, National Aeronautics and Space Administration Johnson Space Center, Houston 77058; and 3Division of Mathematics and 4Department of Physics, University of Houston, Houston, Texas 77004

Received 20 April 2000; accepted in final form 3 August 2000

Foster, Philip P., Alan H. Feivoson, Roland Glowinski, Michael Izygon, and Aladin M. Boriek. A model for influence of exercise on formation and growth of tissue bubbles during altitude decompression. Am J Physiol Regulatory Integrative Comp Physiol 279: R2304–R2316, 2000.—In response to exercise performed before or after altitude decompression, physiological changes are suspected to affect the formation and growth of decompression bubbles. We hypothesized that the work to change the size of a bubble is done by gas pressure gradients in a macro- and microsystem of thermodynamic forces and that the number of bubbles formed through time follows a Poisson process. We modeled the influence of tissue O2 consumption on bubble dynamics in the O2 transport system in series against resistances, from the alveolus to the microsystem containing the bubble and its surrounding tissue shell. Realistic simulations of experimental decompression procedures typical of actual extravehicular activities were obtained. Results suggest that exercise-induced elevation of O2 consumption at altitude leads to bubble persistence in tissues. At the same time, exercise-enhanced perfusion leads to an overall suppression of bubble growth. The total volume of bubbles would be reduced unless increased tissue motion simultaneously raises the rate of bubble formation through cavitation processes, thus maintaining or increasing total bubble volume, despite the exercise.

A bubble is defined as a volume of gas in a tissue that follows the phenomenological laws of ideal gases, diffusion and surface tension. A bubble is formed by the initial explosive bubble growth involving the surrounding tissue (11, 40) and may recruit all dissolved gases. To reduce bubble formation and growth, a denitrogenation or N2 “washout” procedure consisting of prebreathing a hypoxic mixture is performed before ascent to a constant working altitude pressure. We refer to the overall sequence of O2 prebreathe, ascent, and time at altitude as a “decompression profile.” When referring to the actual process of pressure reduction, however, we use the simpler term “decompression.” Bubbles may form, grow, and decay during the sojourn at altitude, usually disappearing on recompression to sea level.

It has become increasingly apparent that skeletal muscle exercise, regardless of when it is performed, influences the onset of DCI (20, 48). A possible explanation is that exercise may create gas micronuclei (44). In particular, high-intensity exercise before decompression may create gas micronuclei (19), which increase the risk of DCI. Also, mechanical movement of body structures may cause cavitation (19) and increase the production of bubbles after decompression (20). Although exercise may accelerate N2 elimination, it does not invariably precipitate bubble formation (20) and, therefore, may even induce a protection against DCI. Experimental results from Webb et al. (48) indicated that moderate exercise performed during the O2-prebreathe period enhanced the tissue N2 washout and reduced the incidence of DCI. Here, we develop a bubble formation-and-growth model (FGM) to answer the following questions. First, how do exercise-induced mechanisms impact formation and growth of gas bubbles? Second, are these mechanisms competing, and if so how? In the accompanying study (11a), the FGM will be validated in a survival analysis to predict the incidence of DCI in the National Aeronautics and Space Administration Altitude Experimental Data Set.

A bubble is defined as a volume of gas in a tissue that follows the phenomenological laws of ideal gases, diffusion and surface tension. Bubble growth is controlled by the classical laws of motion: the pressure of the gas provides the driving force to expand the bubble, while the inertia and elastic recoil of the tissue, to-
gether with the interfacial tension of the bubble wall, provide resistance to expansion (40). The actual work to change the bubble volume is accomplished solely by pressure gradients of gases across the interface between the bubble and surrounding tissue (21). During breathing of pure O\textsubscript{2} after decompression, the N\textsubscript{2} pressure gradient is directed from the tissue to the alveolus. Although this pressure gradient creates a flux that tends to remove N\textsubscript{2} from the vicinity of the tissue bubble, N\textsubscript{2} still diffuses into the bubble, which enlarges (40). In our hypotheses, the relevant system in which thermodynamic forces act consists of two distinct spatial and functional subsystems. First, there is a microsystem consisting of tissue volume containing the bubble, its boundary layer, and a tissue shell. This microsystem then interacts with a macrosystem as the alveolus-arterial blood-tissue shell-venous blood serial cascade of structural or functional barriers. We then developed the FGM to explain how bubble growth is influenced by exercise-induced changes in the O\textsubscript{2} physiological resistances in series in both systems.

Because bona fide bubbles appear to form randomly (47, 50), we hypothesized that their spatial and temporal distributions in small units of tissue volume follow a Poisson process. Informally, the Poisson process asserts that the event of bubble formation occurs independently through time in any of a large number of small units of tissue volume, but with a small probability in any given unit at a given time (32, 38). This process is characterized by parameters that may depend on the type, intensity, duration, and chronology of exercise. At working altitude pressure, the total volume of all bubbles in tissue is propagated in time through the growth-and-decay mechanism, which applies independently for each bubble relative to its time of formation.

We demonstrated the potential mechanisms of exercise by applying the FGM in simulations to calculate total bubble volume for several variations of decompression procedures typical of actual EVAs. These variations were primarily characterized by differences in O\textsubscript{2} consumption, blood flow, and bubble formation rates (Poisson process). The results of our simulations suggested that exercise-induced elevation of O\textsubscript{2} consumption at altitude facilitated the persistence of bubbles in tissues, whereas exercise-enhanced perfusion tended to suppress bubble growth. The total volume of bubbles would be reduced unless increased tissue motion simultaneously raises the rate of bubble formation through cavitation processes, thus maintaining or increasing total bubble volume, despite the exercise.

### Glossary

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>(a−a)\textsubscript{O\textsubscript{2}}(t)</td>
<td>Alveolar-arterial \textsubscript{O\textsubscript{2}} difference, Pa</td>
</tr>
<tr>
<td>\alpha</td>
<td>Parameter of the Poisson process, dimensionless</td>
</tr>
<tr>
<td>\Psi_{O\textsubscript{2}}(t)</td>
<td>Arterial partial tension of dissolved \textsubscript{O\textsubscript{2}} that is not utilized in tissue metabolism, Pa</td>
</tr>
<tr>
<td>\Phi_{O\textsubscript{2}}(t)</td>
<td>Overall \textsubscript{O\textsubscript{2}} pressure gradient in the macro- and microsystem, Pa</td>
</tr>
<tr>
<td>M\textsubscript{b}(t)</td>
<td>Number of gas moles in the bubble, mol</td>
</tr>
<tr>
<td>P\textsubscript{A\textsubscript{i}}(t)</td>
<td>Alveolar partial pressure of gas i, Pa</td>
</tr>
<tr>
<td>P\textsubscript{r}(t)</td>
<td>Tension drop of dissolved \textsubscript{O\textsubscript{2}} due to \textsubscript{O\textsubscript{2}} consumption, Pa</td>
</tr>
<tr>
<td>\nu</td>
<td>Tissue elastic recoil from Ref. 14, 3.7 × 10\textsuperscript{5} Pa (= 3.7 × 10\textsuperscript{5} dyn/cm\textsuperscript{2})</td>
</tr>
<tr>
<td>\beta</td>
<td>Parameter of the Poisson process, dimensionless</td>
</tr>
<tr>
<td>D\textsubscript{i}</td>
<td>Diffusivity of the ith gas species in the tissue, m\textsuperscript{2}/min</td>
</tr>
<tr>
<td>\epsilon</td>
<td>Thickness of the diffusion barrier (protein layer), 2 × 10\textsuperscript{-6} m</td>
</tr>
<tr>
<td>F\textsubscript{IO\textsubscript{2}}</td>
<td>Fraction of \textsubscript{O\textsubscript{2}} in the inspired medium, dimensionless</td>
</tr>
<tr>
<td>h</td>
<td>Constant of proportionality = 2, dimensionless</td>
</tr>
<tr>
<td>\Omega</td>
<td>Constant of proportionality estimated from Table 2, 0.87, dimensionless</td>
</tr>
<tr>
<td>\Psi_{CO\textsubscript{2}}(t)</td>
<td>Number of dissolved moles of \textsubscript{CO\textsubscript{2}} in the tissue, mol</td>
</tr>
<tr>
<td>\Psi_{H\textsubscript{2}O}(t)</td>
<td>Number of moles of water vapor from the tissue shell, mol</td>
</tr>
<tr>
<td>\Psi_{Hb}(t)</td>
<td>Number of gas moles in the tissue shell, mol</td>
</tr>
</tbody>
</table>

### Equations

- \text{Mixed venous } \text{PO}_{2}, P\text{a}_{O\text{2}}(t)
- \text{Partial pressure of gas } i \text{ in inspired breathing medium, Pa}
- \text{Tension of gas } i \text{ in the mixed venous blood, Pa}
- \text{Tension drop of dissolved } \text{O}_{2} \text{ due to } \text{O}_{2} \text{ consumption, Pa}
- \text{Pressure of metabolic gases in the bubble, Pa}
- \text{Partial pressure of gas } i \text{ in the mixed venous blood, Pa}
the tissue element (V_{tot}) to the bubble volume (V_b) is consistent. The part of the tissue element that does not include the bubble per se will be referred to as the homogeneous tissue shell. We proceed to derive a differential equation relating the bubble radius (R_b), and hence V_b, to physical and physiological parameters obtainable from the characteristics of the decompression profile.

We consider two gas transport systems: a macrosystem, in which gases move from the alveolus to the tissue element and vice versa, and a microsystem for gas exchanges across the diffusion barrier inside the tissue element. In the macrosystem, CO_2 moves outward from the tissue element to the alveolus, and this flux is considered in the positive direction. Similarly, after decompression during breathing of N_2-O_2 mixtures, N_2 moves in the positive direction, from the super-saturated tissue element to the alveolus. In contrast, O_2 moves from the alveolus toward the tissue element, and the flux has a negative direction. Finally, water in the tissue fluid also tends to move toward the tissue element (negative direction) as it vaporizes to fill the empty space created by the forming bubble. In the microsystem (bubble-diffusion barrier-tissue shell), we establish signs for the gradient and flux of a gas across the diffusion barrier to be “positive” if it has the same direction as the gas flux in the macrosystem (21, 28).

During the initial explosive bubble growth phase, all gases in this system diffuse into the de novo bubble (11). Thus CO_2 and N_2 have “negative” fluxes in this system, whereas O_2 has a positive flux. Water vapor diffuses into the de novo bubble; hence, it too has a positive flux in this system (Fig. 1B).

**Diffusion of gases across the diffusion barrier in the microsystem.** The volumes of gases are expressed under standard body conditions of temperature, ambient pressure, and saturated with water vapor (BTPS). In a three-dimensional coordinate system, the molar flux of the ith permeating ideal gas species (J_i) obeys Fick’s first phenomenological law of diffusion (4, 14, 17, 18, 33, 39, 43) and can then be estimated by J_i = (-RT/D_i)\partial{P_i}/\partial{x}, where P is the ambient pressure, R is the universal gas constant, T is the temperature in degrees Kelvin, D_i is the diffusion coefficient, and \partial{x} is the molar fraction gradient of the ith gas in the macrosystem. For air breathing, gas exchange dynamics involve four relevant species: CO_2, N_2, O_2, and H_2O (i = 1,...,4). In the macrosystem, \partial{x} is defined along a direct path to the center of the bubble, where the inward direction is negative and outward is positive. Resulting fluxes have signs in accordance with the “macrosystem” rule (inward = negative; outward = positive). In the microsystem, we establish that a flux is positive if it has the same direction as in the macrosystem (opposite direction = negative). Therefore, the CO_2 and N_2 fluxes are negative (Fig. 1B). The net flux (28) (J_N) into or out of the bubble is expressed as follows

\[
J_N = -\frac{P}{RT} \sum_{i=1}^{4} D_i \frac{\partial}{\partial x_i}
\]

Applying Henry’s law to the dissolved tissue gases surrounding the bubble and using the ideal gas equations for gas pressures inside the bubble (14), J_N(t) can be approximately expressed in terms of partial pressures of the ith gas as a function of time. As reported previously (14, 28), the net flux is thus expressed as follows

\[
J_N(t) = -\frac{1}{RT} \sum_{i=1}^{4} D_i [s_{i,\text{ti}}(t) - P_{i,\text{ti}}(t)]
\]

where s_{i,\text{ti}} is the solubility of the ith gas species in the tissue element, \epsilon is the thickness of the diffusion barrier, and P_{i,\text{ti}} and P_{i,\text{tu}} are the partial tissue tension and pressure within the bubble of the ith gas, respectively. When referring to a specific gas species, we use the convention of replacing the subscript i by the gas name (e.g., s_{i,O_2} instead of s_{i,N_2}). Time is
expressed in minutes and measured from the start of the prebreathe period to the end of the exposure to altitude. The prebreathe period begins at time \( t = t_0 = 0 \), when partial pressures of gases in the breathing medium start to change from the equilibrium of standard atmospheric conditions. Arrival at working altitude pressure (end of depressurization), occurs at \( t = t_{alt} \).

Moles of gas within the tissue shell and in the bubble. To evaluate the net flux (Eq. 2), we next calculate the number of moles of each gas crossing the diffusion barrier. Suppose a particular bubble forms at \( t = t_0 \), let \( M_{t0}(t) \) be the total number of moles of gas in the tissue shell. From Henry’s law (14), we have

\[
M_{t0}(t) = \frac{s_{\text{N}_2} V_{\text{i}}(t) P_{\text{N}_2}(t) + s_{\text{CO}_2} V_{\text{i}}(t) P_{\text{CO}_2}(t)}{RT} + m_{\text{CO}_2}(t) + m_{\text{H}_2\text{O}}(t)
\]

where \( m_{\text{CO}_2}(t) \) and \( m_{\text{H}_2\text{O}}(t) \) are the number of moles of dissolved \( \text{CO}_2 \) and water vapor, respectively, \( V_{\text{i}}(t) \) is the tissue shell volume pertaining to the bubble at time \( t \) [which has been in existence for \( t = t_0 \) min], and \( \Phi_{\text{O}_2}(t) \) is the \( \text{O}_2 \) overall pressure difference between the macro- and the microsystem. Using the equation of state of an ideal gas (14), we estimate the number of moles of gas in the bubble

\[
M_b(t) = \frac{V_b(t)}{RT} [P_{\text{N}_2}(t) + P_{\text{CO}_2}(t) + P_{\text{O}_2}(t) + P_{\text{H}_2\text{O}}(t)]
\]

where \( V_b(t) \) is the volume of the bubble at time \( t \).

Estimation of the bubble radius. From the law of conservation of mass (21, 28), the number of moles diffusing into and out of the bubble per minute is

\[
A_{b}(t) J_{b}(t) = M_{b}(t) - M_{t}(t)
\]

for \( t > t_0 \), where \( A_{b}(t) \) is the surface area of the bubble and \( M_{b}(t) \) and \( M_{t}(t) \) are the gas mole uptake into and out of the tissue shell (microsystem) and into and out of the bubble, respectively. (We use the convention that the overdot denotes differentiation with respect to time.) By definition of the microsystem, we assume \( V_{\text{i}} \) to be proportional to \( V_{\text{b}} (h = V_{\text{i}}/V_{\text{b}} \geq 1) \) and \( h \) to be sufficiently small so that any gases entering or leaving the tissue shell are being involved in exchanges across the diffusion barrier of the microsystem. Finally, for \( t > t_0 \), Eq. 5 can be rewritten in terms of the bubble radius \( R_b(t) \)

\[
\dot{R}_b(t) = \frac{J(t) - \frac{1}{2} \dot{R}_b(t) [hL(t) - \dot{K}(t)]}{hL(t) - \dot{K}(t)}
\]

where \( L(t) \) and \( K(t) \) are quantities derived in the Appendix. For a given decompression profile, values of \( K(t) \) and \( L(t) \) may be determined by a function of time through measurements of inspired pressure and fraction of gases. Details of the calculation of \( K(t) \) and \( L(t) \) are given in the Appendix. Equation 6 has no analytic solution for \( R_b(t) \) and must be solved numerically.

Gas transport in the macrosystem: estimation of pressures and/or tensions. To obtain \( L(t) \) and \( K(t) \) in Eq. 6, it is first necessary to estimate the pressure gradients in the macrosystem. These may be calculated from values of the absolute pressure and inspired fractions of \( \text{N}_2 \) and \( \text{O}_2 \) and expired \( \text{CO}_2 \) in the breathing medium for each phase of our decompression profiles. In addition, within the macrosystem, we consider partial pressures of each gas: inside the alveolus \( [P_a(t)] \), inside the pulmonary capillary \( [P_{\text{c}}(t)] \), and in the mixed venous blood \( [P_{\text{v}}(t)] \). For all gases, we assume \( P_{\text{v}}(t) = P_{\text{t}}(t) \).

\( \text{N}_2 \) tissue tension. On the downstream side of the macrosystem flow, we estimated \( P_{\text{t}}(t) \) for \( t > t_{\text{alt}} \) using the classical exponential equation (8, 11, 14). Using this method, we computed \( P_{\text{t}}(t) \) by increments at fixed times \( t_1, t_2, \ldots, t_n \), where \( t_{n-1} = t_{\text{alt}} \) and \( t_n = t \). The incremental expression for \( P_{\text{t}}(t) \) is given by

\[
P_{\text{t}}(t) = [P_{\text{a}}(t) + [P_{\text{t}}(t_{n-1}) - P_{\text{a}}(t_{n-1})]e^{-k_2(t-t_{n-1})}]
\]

where \( P_{\text{a}}(t) \) is the \( \text{N}_2 \) arterial tension derived from the alveolar gas equations and partial pressures (11, 42) and \( k_2 \) is the tissue gas exchange rate constant for \( \text{N}_2 \).

\( \text{O}_2 \) transport. On the upstream side of the macrosystem flow portrayed in Fig. 1B, \( \text{O}_2 \) is driven through a series of interfaces (12) into the tissue element, where it is dissolved. To correctly model the flow, it is necessary to estimate pressure differences across various encountered interfaces as follows (Fig. 1C). First, because of the alveolar membrane, there is an alveolar-arterial pressure difference \( [P_{\text{a}} - P_{\text{a}}(t)] \), which tends to increase with \( P_{\text{a}}(t) \) (5, 15, 37). On the basis of the tissue gas exchange (14), we estimate (a-al)\( P_{\text{a}}(t) \) with an algorithm that uses values given by Clark and Lamberts (5). Then we obtained \( P_{\text{a}}(t) \) by subtraction from \( P_{\text{a}}(t) \). Second, the \( \text{O}_2 \) transfer to the capillaries occurs when \( \text{O}_2 \) is transported in physical solution or bound to Hb. Above the threshold \( P_{\text{a}}(t) \) of 13.33 kPa (100 Torr, 1 kPa = 7.50062 Torr), the \( \text{O}_2 \) saturation is assumed to be 100% and the unbound \( \text{O}_2 \) remains in physical solution. In terms of tension, the arterial \( \text{O}_2 \) tension is made up of two components, the \( \text{O}_2 \) dissolved fraction \( \text{(alveolar)} \) and the Hb-bound \( \text{O}_2 \) \( \text{(arteriovenous)} \). We used an algorithm derived from the \( \text{O}_2 \) dissociation curve (29, 35, 36) to estimate \( P_{\text{Hb}}(t) \) as a function of \( P_{\text{Hb}}(t) \). Third, the \( \text{O}_2 \) supply to the mitochondria results in the withdrawal of \( \text{O}_2 \) from further utilization in the \( \text{O}_2 \) transport process across the bubble boundary layer. Di Prampero and Ferretti (9, 10) derived a relationship between the \( \text{O}_2 \) tissue consumption \( [V_{\text{t}}(t)] \) and the arteriovenous tension difference \( (\text{alveolar}) \) and the arteriovenous \( \text{O}_2 \) difference \( (\text{arteriovenous}) \). The relationship is

\[
V_{\text{t}}(t) = \frac{P_{\text{Hb}}(t)}{R_b(t) \cdot P_{\text{Hb}}(t) R_{\text{c}}(t) \cdot P_{\text{Hb}}(t) R_{\text{c}}(t)}
\]

where \( R_{\text{c}}(t) \) is the circulatory convective resistance and \( P_{\text{Hb}}(t) \) is the tension drop of dissolved tissue \( \text{O}_2 \) due to \( \text{O}_2 \) consumption. Finally, \( \text{O}_2 \) supplied to the tissue element can potentially participate in bubble gas exchanges of the microsystem. Thus the tension of the dissolved \( \text{O}_2 \) that is applied to the microsystem at time \( t \) \( [P_{\text{t}}(t)] \) can be written in the form

\[
P_{\text{t}}(t) = P_{\text{Hb}}(t) - P_{\text{Hb}}(t) - P_{\text{v}}(t)
\]

Dissolution of \( \text{O}_2 \) in the tissue element follows Henry’s law and is described by an exponential relation similar to Eq. 7. As was the case for \( \text{N}_2 \), \( \Phi_{\text{O}_2}(t) \), the overall \( \text{O}_2 \) pressure gradient in the macro- and microsystem that applies from the alveolus to the tissue element can be expressed incrementally at fixed times \( t_1, t_2, \ldots, t_n = t \), so that

\[
\Phi_{\text{O}_2}(t) = \Phi_{\text{O}_2}(t) + (1 - a)\Phi_{\text{O}_2}(t)
\]

where \( k_2 \) is the tissue gas exchange rate constant for \( \text{O}_2 \). Indeed, metabolism lowers the \( \text{O}_2 \) tension in the tissue element below \( P_{\text{a}}(t) \), creating a phenomenon known as the “\( \text{O}_2 \) window” or “inherent unsaturation” (11, 16, 23, 34, 42).
Equations 7 and 9 are then used in the APPENDIX to calculate $L(t)$ and $K(t)$.

Number and onset times of bubbles in tissue. Youn (50) proposed a stochastic model for accretion and deletion of skin molecules that leads to an exponential distribution for the number of micronuclei. Here, we consider a stochastic model of bubble formation in which micronuclei evolve into bubbles at random times after decompression following a Poisson process. Consider a tissue region divided into $n$ units of tissue that are identical in makeup and perfusion (40). Each tissue unit is independent, and there is no diffusion from one unit to another. All tissue units are the same size, and units are evenly distributed in space. In contrast, bubbles in one unit are of different age and size. Schematically, the units are illustrated as cubes of constant volume $V_{in}$ in Fig. 1D. For a total time at altitude of $T$ minutes, $N_i(t)$ (i = 1, . . . , n), the number of bubbles formed in the $i$th unit of tissue volume up to time $t$ (in minutes, $0 < t < T$) is assumed to follow a nonhomogeneous Poisson process (32, 38) with intensity $v(t)$, $v(t) = \alpha e^{-\beta t}$. The parameters $\alpha$ and $\beta$ are driven by the decompression procedure and the level of exercise or rest. For a given procedure, $\alpha$ also serves as a scaling factor, being proportional to $V_{in}$. The decreasing exponential form of $v(t)$ reflects the rapidly decreasing propensity to form bubbles as exposure time increases (6, 22).

To be a Poisson process, $N_i(0) = 0$, $N_i(t)$ must have independent increments; i.e., the number of bubbles formed in nonoverlapping intervals of time must be statistically independent, and two or more bubbles cannot form simultaneously. Physically, the latter two requirements correspond to temporal and spatial independence of bubble formation. It can be shown under these assumptions (32) that $N_i(t)$ has a Poisson distribution with mean

$$m(t) = \int_0^t v(u)du = \frac{\alpha}{\beta} (1 - e^{-\beta t}) \quad (10)$$

In other words, the probability $p$ to obtain $k$ bubbles up to time $t$ is

$$p[N_i(t) = k] = \exp[-m(t)][m(t)]^k/k! \quad (k = 0, 1, . . .).$$

Mean bubble radius and total volume of bubbles. The mean bubble radius for the given region of tissue at time $t$ can be written as

$$R_{b,j}(t) = \frac{1}{N(t)} \sum_{i=1}^{n} \sum_{j=1}^{N_i(t)} R_{b,ij}(t) \quad (11)$$

where $N(t) = \sum_{i=1}^{n} N_i(t)$ and $R_{b,ij}(t)$ is the radius of the $j$th bubble in the $i$th tissue unit. By convention, we take $R_{b,ij}(t) = 0$ if the bubble has not formed by time $t$. Under the additional assumption that bubbles form and grow independently over the $n$ tissue units, $N(t)$ is also a Poisson process (32, 38). If the region is homogeneous in the sense that all the Poisson processes $N_i(t)$ have identical values of $\alpha$ and $\beta$, then the mean of $N(t)$ is simply $n(\alpha/\beta)(1 - e^{-\beta t})$. With the assumption of spherical bubbles, the total volume of bubbles in the region of tissue at time $t$ is

$$V_b(t) = \frac{4}{3} \pi \sum_{i=1}^{n} \sum_{j=1}^{N_i(t)} R_{b,ij}^3(t) \quad (12)$$

**Experimental Design**

We applied the FGM to four types of decompression profiles (A–D) typical of chamber tests and actual EVAs (Table 1). These profiles shared the following properties: 1) the duration of prebreathe was 210 min (P = 101.13 kPa, FIO$_2$ = 1; 2) ascent time was 6 min; and 3) FIO$_2$ = 1. The altitude pressure was 30 kPa for profiles A–C compared with 60 kPa for profile D. In profiles A and D, physiological parameters $P_{V_{1/2},O_2}, R$, $t_{1/2,N_2}$, and $t_{1/2,CO_2}$ were set to known values of 8 kPa, 0.82, 360 min (7), and 313.2 min, respectively. These values are consistent with the case of no exercise. In addition, $t_{1/2,CO_2}$ was calculated using Eq. A13. Using experimental data, we found in another study (11a) that the Poisson process parameter $\beta = 0.017$ for profiles without prebreathe exercise but with mild exercise (817 kJ) at altitude. We assumed that, for a control case of no exercise at any time, $\beta$ would be reduced by 20% (to $0.014$), reflecting the decreased propensity to form bubbles. In contrast, profile B incorporated mild exercise at altitude to emulate the moderate workloads performed by astronauts during ordinary EVA. For this case, we assumed $P_{V_{1/2},O_2} = 10$ kPa, $R = 0.95$, $t_{1/2,N_2} = 200$ min, and $t_{1/2,CO_2} = 174$ min. Profile C also simulates exercise, but with two phases (10 min of heavy exercise followed by 25 min of light exercise) during prebreathe. Physiological parameters were set at $P_{V_{1/2},O_2} = 12$ kPa, $R = 1.12$, $t_{1/2,N_2} = 60$ min, and $t_{1/2,CO_2} = 52.2$ min (phase 1) and then at $P_{V_{1/2},O_2} = 10$ kPa, $R = 0.95$, $t_{1/2,N_2} = 80$ min, and $t_{1/2,CO_2} = 69.5$ min.

For each profile type, various simulations of bubble formation and growth were analyzed to examine the effect of exercise on $R_{b}(t)$ and $V_b(t)$. For simulations A9–A11 (Table 1), we modified values of $P_{V_{1/2},O_2}$ while keeping all other parameters fixed to isolate the effect of $V_{ti,O_2}$. These higher values are representative of heavier exercise workloads required to perform EVA tasks during some orbital missions. For simulations A7 and A8, $\beta$ was increased by a factor of 3 to emulate...
For simulations involving cavitation (19) and increase the bubble formation rate (20).

Conjectured that mechanical motion of tissues may cause without significant additional O₂ consumption. It has been this would be the case if mechanical motion takes place physiological parameters remaining at no-exercise levels.

The effect of an increased rate of bubble formation, but with physiological parameters remaining at no-exercise levels. This would be the case if mechanical motion takes place cause cavitation (19) and increase the bubble formation rate (20). For simulations involving profiles A, C, and D, the parameter α was chosen to make the expected number of bubbles per tissue unit (≈ α/β) equal to 6.0 over 50 tissue units.

For the case of exercise at altitude (simulations B1–B14), we investigated the effect of bubble formation rate on maximum bubble volume. This rate was again controlled by varying β. During this exercise, more units of tissue would be recruited for bubble formation; therefore, we increased n, the number of tissue units, to 100. Also, the density of bubbles per tissue unit would be expected to be greater than at rest; hence, we changed α so that the mean number of bubbles per unit was 25.

Simulation Process

An overview of the simulation is illustrated in Fig. 2. Working values of solubilities of gases (N₂, O₂, and CO₂) in the tissue element were chosen to lie in the range of similar values for blood (Table 2). Diffusivities of gases were chosen as ~75% of corresponding values for water (Table 2) and 200% of the values for lipids. Values of other physical constants are listed in the Glossary.

From the Poisson distribution with mean m(t) (Eq. 10), we generated Nₐ(t), the total number of bubbles formed over a decompression period of T minutes. Values of α and β defining m(t) are given in Table 1. Methodology for generating random numbers from the Poisson distribution is well known (32). Next, we used a property of Poisson processes (32) that relates the conditional distribution of event times to the random numbers from the Poisson distribution is well known (32). Next, we used a property of Poisson processes (32) that relates the conditional distribution of event times to the

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<th>t₁/₂,O₂, min</th>
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<th>β</th>
<th>Rb, max, µm</th>
<th>Vb, max, mm³</th>
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*SOME parameters have values of exercise at altitude. See Glossary for definitions of abbreviations.
Adams or stiff Gear, Fehlberg order 4–5, or Runge Kutta methods for nonstiff equations was required to achieve convergence. The Gel'fand-Lokutsiyevski chasing method was used for solving the boundary value problem. Processing of 300 bubbles took ~2 min to run on a 300-mHz personal computer with 64 MB of random access memory. Finally, the R_{b,i}(t) was used in Eqs. 11 and 12 to calculate R_{b,max} and V_{b,z}(t).

RESULTS AND DISCUSSION

Effects of O_2 Consumption on Bubble Dynamics

Simulations A1 and A9–A11 were run with different levels of O_2 consumption at altitude (P_{vO_2} = 8, 12, 18, and 20 kPa) with bubble formation parameters (α and β) and blood flow fixed at resting control values (Table 1). Bubble growth to R_{b,max} was essentially the same for all four levels of O_2 consumption. These results are consistent with those of Van Liew et al. (42), who noted that a large O_2 window, especially during O_2 breathing, reduced the bubble enlargement. They also reported no significant change in the O_2 window values at arteriovenous pressure differences, 100 kPa (FIO_2 = 1). Similarly, we observed that increasing O_2 extraction (P_{vO_2} > 9 kPa) had little or no effect on the O_2 window, and therefore maximal bubble growth was not affected. However, the average bubble size decreased slowly in time when V_{O_2} increased, whereas low resting V_{O_2} values facilitated faster decay (Fig. 3A). Because O_2 has a greater permeation coefficient than N_2, short transients of O_2 permeate rapidly into the bubble at rest [P_{vO_2}(t) = 8 kPa]; simultaneously, N_2 exits the bubble to the surrounding tissue (40). Then, O_2 rapidly permeates out of the bubble, resulting finally in a rapid bubble decay. In contrast, tissue O_2 extraction is enhanced during exercise; thus a relatively small amount of O_2 diffuses into the bubble and is exchanged for N_2. Therefore, N_2 builds up in the bubble, which in turn reduces the bubble decay rate.

Exercise Decreases the Mean Bubble Radius

Aerobic exercise-enhanced blood flow, before and/or after decompression, generates a cascade of events in the macro- and microsystem as follows. Augmentation of blood flow [Q_{ti}(t); Eq. A12] causes a decrease in the O_2 and N_2 tissue washin and washout half times, t_{1/2,i} = (ln 2)/k_i, where k_i is the tissue gas exchange rate constant. As a result, excess N_2 in the tissue element is carried away before it can diffuse into postdecompression bubbles. The fast N_2 removal by blood precludes bubble enlargement (40). Also, little or no N_2 is carried to the tissue when breathing enriched O_2 mixtures. In contrast, a greater amount of O_2 physically dissolves in the tissue element, which in turn moderately reduces the O_2 window.

Hills and LeMessurier (16) reported that, after 15 min of exposure of rabbits to an hyperoxic medium, the O_2 window was large. Here, we agree that only a small amount of O_2 would dissolve in the tissue during this time interval. However, during several hours of hyperoxic O_2 prebreathe/altitude exposure, greater amounts of O_2 would dissolve in tissues, thereby eventually reducing the O_2 window. According to Lambertsen et
al. (23), other mechanisms such as hypercapnic vasodilatation may also facilitate the O2 transfer to the tissue, which in turn accelerates the window reduction. Lambertsen et al. also showed in humans, that arterial hypercapnia, a highly potent vasodilator of cerebral blood vessels, in conjunction with breathing for 15 min at 3.5 atm (fraction of CO2 in the inspired medium 5 0.02, FIO2 5 0.98) induced a significant elevation of PV2 (146 kPa) in internal jugular venous blood samples, as compared with breathing without hypercapnia (FIO2 5 1, PV2 5 10.13 kPa). In both cases Pao2 5 266 kPa; however, hypercapnia via cerebral vasodilatation reduced the O2 window in very limited time.

Observations in pigs breathing hyperoxic mixtures showed that the bubble incidence in the pulmonary artery was reduced as Pao2 increased (34). Here, for pure O2 breathing, we illustrate how P 5 Pao2 affects Rb(t) through change in PaO2. Figure 3B shows Rb(t) for the case of no exercise at two different altitudes: P 5 30 kPa (simulation A1) and P 5 60 kPa (simulation D1). At 60 kPa, the larger value of Pao2 induces a significant reduction in Rb(max). Therefore, we postulate that the O2 window inhibits bubble enlargement at the beginning of the altitude exposure. Later, because of increased perfusion, the higher O2 flow to tissues augments the amount of dissolved O2 in the unit of tissue volume. This, in turn, decreases the O2 window, thus tending to slow the rate of bubble decay. However, the greater amounts of physically dissolved O2 facilitate increased O2 exchange within the microsystem. According to Van Liew and Burkard (41), many short O2 transients would permeate rapidly in the bubble, resulting finally in a marked decay rate of bubble radii.

Figure 4 compares Rb,max for decompression profiles whose physiological parameters (PV2, , R, t1/2,N2, and t1/2,O2) have the characteristics of no exercise, moderate exercise at altitude, and heavy exercise during prebreathe, but where is held fixed. First, we compared the effect of exercise when is set to the resting value of 0.014 (simulations A1, B1, and C1). Then a similar comparison was made for 5 0.017 (simulations A2, B2, and C2). For both values of , we observed that Rb,max dropped by 5 64% (from 34.1 to 12.4 for 5 0.014 and from 44.5 to 16.2 for 5 0.017). When the effect of heavy exercise during prebreathe was compared with the case where there is no exercise, the drop was more dramatic, i.e., 5 83% for both values of . Results of these simulations illustrate how an augmentation of blood flow, which decreases t1/2,N2 and t1/2,O2, is paralleled by a reduction of bubble radius.

Fig. 3. Simulations of Rb in various conditions. Time is measured from the start of prebreathe period and the elapsed time until altitude exposure begins at 216 min. Some bubbles start to grow on arrival at altitude. A: simulations of decompression profile A under 4 levels of O2 consumption, i.e., PV2 at 8 kPa (A1), 12 kPa (A9), 18 kPa (A10), and 20 kPa (A11); as Vt2 (PV2) increases, bubbles decay less rapidly. B: effect of the O2 window. At an altitude exposure of 60 kPa (D1, FIO2 5 1), a large O2 window suppresses bubble growth. Two realizations (A1 and A3) of profile A (no exercise) show very little difference in Rb; variance due to the Poisson process is minimal and does not significantly impact interpretation of the results. See Glossary for definition of abbreviations.

Fig. 4. Maximum bubble growth in various conditions of rest and exercise. Simulations of no exercise (A1, A2, A7, and A8), exercise at altitude (B1 and B2), and heavy exercise during prebreathe (C1 and C2) are shown.
regardless of the intensity of the Poisson process and the level of \( V_{\text{b,rest}} \).

Also, with an acceleration of the nucleation process (\( \beta \) increased from 0.014 to 0.017), the increase in \( R_{b,max} \) was much more pronounced for the no-exercise case than for the case of exercise at altitude. However, the relative increase was about the same (30%), and for the prebreathe case the relative change appeared less (~20%). This is probably because, regardless of the nucleation rate, only small amounts of dissolved \( N_2 \) remain to be available for bubble growth after the start of decompression.

For profile \( A \), we investigated the effect of increasing \( \beta \) from its resting value of 0.014 to large values reflecting intense bubble formation with physiological parameters remaining fixed (\( \bar{P}_{O_2} = 8 \) kPa, \( R = 0.82 \), \( t_{1/2,N_2} = 360 \) min and \( t_{1/2,O_2} = 313.2 \) min). We observed that in simulations \( A1, A2, A7, \) and \( A8 \), \( R_{b,max} \) increased from 34.1 to 49.8 \( \mu \text{m} \) as \( \beta \) increased from 0.014 to 0.051 (Fig. 4). As \( \beta \) increases, early formation of bubbles before \( N_2 \) supersaturation drops, leading to larger radii than the resting case (simulation \( A1 \)). The pattern of increase was nonlinear, tending toward a plateau for large values of \( \beta \). This is because, no matter how early the bubble is formed, for a fixed \( N_2 \) pressure gradient, the amount of \( N_2 \) available for diffusion into a bubble is limited.

Does Exercise at Altitude Increase the Volume of Tissue Bubbles?

In general, it has been observed that there is an increased incidence of DCI symptoms for subjects exercising at altitude after decompression (20). However, a diving experiment showed that exercise during decompression actually reduced Doppler-detectable venous gas emboli (19). Therefore, it is unclear how exercise affects the incidence of tissue bubbles. Using the FGM, we evaluated the effect of exercise in terms of how much the bubble formation process would have to be accelerated to achieve a value of \( V_{b,max} \) equal to that in a no-exercise case. For example, simulations \( A5 \) (no exercise) and \( B5 \) (exercise at altitude) produced about the same value of \( V_{b,max} \) (0.049 \( \text{mm}^3 \)). In the latter case, \( \beta \) had to be increased by a factor of \(~2.5 \) (from 0.014 to 0.38) to achieve the same \( V_{b,max} \). In other words, ~9.5 times as many bubbles would have to be generated with exercise at altitude to achieve the same maximum volume as at rest. Despite similar values of \( V_{b,max} \), \( V_{b,t}(t) \) differed considerably. For the exercise case (simulation \( B5 \)), a relatively large number of smaller bubbles formed earlier, whereas in the no-exercise case (simulation \( A5 \)), larger bubbles were formed, but mostly at later times (Fig. 5A).

As demonstrated in Fig. 4, exercise-enhanced blood flow reduces \( R_{b,max} \) and, therefore, \( V_{b,max} \) for a fixed number of bubbles formed. In this case, the only way \( V_{b,max} \) could be increased is through a more intense generation of bubbles. In simulations \( B3–B14 \), we calculated \( V_{b,max} \) for simulation \( B \) (exercise at altitude), with \( \beta \) ranging from 0.036 to 1.5. In Fig. 5B, \( V_{b,max} \) increases rapidly as a function of \( \beta \) when \( \beta \) is small and reaches a plateau of ~0.1 \( \text{mm}^3 \) for \( \beta > 0.3 \). The reason is that, similar to the no-exercise case, the \( N_2 \) pressure gradient limits the supply of \( N_2 \) available for bubble growth in the tissue region.

Stability

We found that varying the initial minimal value of the bubble radius, within a range of 10\(^{-8}\)–10\(^{-4}\) m, did not affect the bubble growth dynamics; thus it appears that the differential Eq. 6 is stable, and therefore, our model is robust with respect to the choice of the size of micronecels. Also, the variability (<2.3%) inherent in the Poisson process did not affect the reproducibility of

Fig. 5. \( A \): effect of exercise at altitude on total volume of bubbles. \( A5 \), no exercise; \( B5 \), exercise at altitude. To achieve the same \( V_{b,max} \) as at rest (\( N(t) \sim 274 \)), \( \beta \) had to be increased by a factor of ~2.5 (from 0.014 to 0.38), resulting in ~9.5 times as many bubbles as with exercise at altitude (\( N(t) = 2,612 \)). For \( B5 \), a relatively large number of smaller bubbles formed earlier; for \( A5 \), larger bubbles were formed, but mostly at later times. Therefore, \( V_{b,max} \) for \( A5 \) was shifted to the right. \( B \): limitation of the maximal bubble volume produced in the tissue region. In simulations \( B3–B14 \), we calculated \( V_{b,max} \) for profile \( B \) (exercise at altitude), with \( \beta \) ranging from 0.036 to 1.5. \( V_{b,max} \) increases rapidly as a function of \( \beta \) when \( \beta \) is small and reaches a plateau of ~0.1 \( \text{mm}^3 \) for \( \beta > 0.3 \). \( \beta \) had to be increased by a factor of ~11.7 (\( \beta = 0.2 \)) in simulation \( B12 \) to achieve the same \( V_{b,max} \) as in simulation \( A6 \) (\( \beta = 0.017 \)).
the simulation results for various realizations of the same profile. Simulations A1 and A3 represent two realizations of profile A (no exercise). As shown in Fig. 3B, there is very little difference in $R_b(t)$ between these two examples. As a further illustration of variability, we found that $R_{b,max}$ varied with a standard deviation of $\sim 1.6\%$ over 13 similar realizations. Also, the time of maximum bubble radius varied with a standard deviation of $\sim 3\%$. However, this small amount of variation does not explain the different decompression outcomes observed over a population of subjects. Significant variability may be associated with age, body mass index, time of the day, seasonal variation, body temperature, body chemistry, and previous injuries (2).

**Review of Assumptions**

Physical parameters, e.g., $s_{i,j}$, $D_i$, $v$, $\tau$, $\epsilon$, and $h$, were selected only for a hypothetical tissue derived as a mixture of known values (3, 4, 14, 24, 41, 46) for blood and lipids. In reality, values of these parameters would be expected to vary for a given subject over time and also between subjects. In addition, the site of formation of critical tissue bubbles is unclear. Not only is there a lack of knowledge about where damaging bubbles are located in the body and where they arise (42), but there are also uncertainties inherent in our calculations. Several simplifications were made as follows. First, the tissue was assumed to be perfused by an infinite number of infinitesimally small capillaries (17, 18), and the tissue element was assumed to be well stirred. The overall $O_2$ pressure gradient in the macro- and microsystem [$\Phi_{O_2}(t)$] was considered homogeneous in the simulated large population of tissue elements/bubbles. Second, we assumed a resting value of $t_{1/2,N_2}$ of 360 min, which has been well documented for “standard” NASA and US Air Force altitude exposures (6–8). We then obtained a corresponding value of $t_{1/2,O_2}$ (313.2 min), through a derived relationship to $t_{1/2,N_2}$ (see Eq. A13). Third, blood flow in tissues and exercising skeletal muscles has not yet been measured during altitude. We therefore approximated $t_{1/2,N_2}$ and $t_{1/2,O_2}$ using a rough linear relationship between $O_2$ consumption and corresponding hypothetical blood flow (1, 25, 26) for an average subject performing mild, moderate, and heavy exercise. Furthermore, physical properties of the tissue such as the overall solubility are modified by exercise-induced tissue hyperemia. Therefore, Eq. A13 should be modified with adjustments of blood flow with exercise along with the solubility of the tissue. However, we were unable to assess the change of tissue solubility with exercise. Fourth, we neglected the possible effects of the expected local temperature rise ($1–2^\circ C$) in critical tissues during the simulated submaximal exercises. From the equation of state of an ideal gas, this minor change of $\approx 0.64\%$ on the Kelvin scale would have negligible effect on the bubble volume. With this small temperature rise, physical constants such as solubilities and diffusivities of gases in tissues would also be expected to remain nearly constant (24). Fifth, even though the metabolic production of $CO_2$ may increase at the onset of our simulated submaximal aerobic exercise, we assumed that the dissolved $CO_2$ tissue tension ($P_{ti,CO_2}$) remains approximately constant during the ensuing steady-state phase due to the concurrent decrease in arterial $CO_2$ tension ($P_{A,CO_2}$) (27). Sixth, the effects of gas diffusion and coalescence between bubbles within a unit of tissue volume were neglected. Seventh, an increase of intra muscular interstitial pressure during skeletal muscle contractions, which may affect the tissue elastic recoil ($v$) and bubble growth, was not considered in our study.

**Perspectives**

Our simulations suggest that exercise-induced elevation of $O_2$ consumption at altitude leads to bubble persistence in tissues. At the same time, however, exercise-enhanced perfusion leads to an overall suppression of bubble growth. The total volume of bubbles would be reduced unless increased tissue motion simultaneously raises the rate of bubble formation (larger values of the Poisson process parameter $\beta$) through cavitation processes, thus maintaining or increasing total bubble volume, despite the exercise. Whether the rate of bubble formation and the incidence of DCI are associated with specific types of mechanical movement of body structures (19) remains to be investigated. Furthermore, measurements of cardiac output and local blood flows in skeletal muscles of a limb, presently under way, will provide further insight into the effects of exercise-induced circulatory changes, particularly during the $O_2$ prebreathe, on the incidence of altitude bubbles and DCI.

**APPENDIX**

The appendix introduces the intermediate steps necessary before calculation of $R_b(t)$ using Eq. 6. After a partial $O_2$ pressure change, mixing of inspired $O_2$-enriched mixtures with lung resident gas is assumed to be approximately complete within 1 min. $P_{AO_2}$ and $P_{AN_2}$ are estimated from the alveolar gas equations (11, 42).

**Calculation of $L(t)$ and $L'(t)$**. Various homeostatic processes maintain $m_{CO_2}(t)$ and $m_{H_2O}(t)$ relatively constant; hence, these two terms can be neglected in the differentiation of $M_i(t)$ and $M_b(t)$ in Eq. 5. When Eqs. 2–5 are combined, the quantity

$$L(t) = \frac{1}{RT} [s_{i,N_2}P_{ti,N_2}(t) + s_{i,O_2}\Phi_{O_2}(t)], \ t > t_b \quad (A1)$$

and its time derivative $L'(t)$ occur explicitly in the expression for $R_b(t)$ given by Eq. 6. To obtain $L'(t)$, we used Eqs. 7 and 9 to calculate

$$\dot{P}_{ti,N_2}(t) = -k_1[P_{ti,N_2}(t_b) - P_{A,N_2}(t)]e^{-k_2(t - t_b)} \quad (A2)$$

$$\dot{\Phi}_{O_2}(t) = -k_3[\Phi_{O_2}(t) + \Psi_{O_2}(t) + (A - a)P_{O_2}(t)]e^{-k_2(t - t_b)} \quad (A3)$$

**Calculation of $K(t)$ and $\dot{K}(t)$**. The expression

$$K(t) = \frac{1}{RT} [P_{b,N_2}(t) + P_{b,CO_2}(t) + P_{b,O_2}(t) + P_{b,H_2O}(t)], \ t > t_b \quad (A4)$$

also occurs in the derivation of Eq. 6. Dissolved gases surrounding the bubble add to the surface tension pressure and
to the elastic recoil of the tissue, which resists expansion. With the use of the Laplace law (4, 13, 14) and application to all dissolved gases, $P_{b,N_2}(t)$, part of the expression of $K(t)$, may be expressed as follows

$$P_{b,N_2}(t) = \Phi_{O_2}(t) + Pt_{i}N_2(t) + Pt_{i}CO_2(t) + Pt_{i}H_2O(t) + \varphi_i(t) - P_{b,m}(t) \tag{A5}$$

In Eq. A5, $\varphi_i(t)$ is the sum of pressures due to surface tension and tissue elastic recoil. With the assumption of perfect spherical bubble, $\varphi_i(t)$ can be expressed in terms of the radius $R_b(t)$

$$\varphi_i(t) = \frac{2\tau}{R_b(t)} + \frac{4}{3}\pi R_b(t)^2 v \tag{A6}$$

where $\tau$ is the surface tension at the bubble-tissue interface and $v$ is the tissue elastic recoil. The last term in Eq. A5, total pressure in the bubble due to metabolic gases, is expressed in terms of its components

$$P_{b,m}(t) = P_{b,CO_2}(t) + P_{b,O_2}(t) + P_{b,H_2O}(t) \tag{A7}$$

By use of Eqs. A5–A7, it follows that

$$K(t) = \frac{1}{RT} \left[ \Phi_{O_2}(t) + Pt_{i}H_2O + \varphi_i(t) \right] \tag{A8}$$

where $\varphi_i(t)$ is readily obtained from Eq. A6. Note that $Pt_{i}CO_2$ and $Pt_{i}H_2O$, being constant, do not contribute to $K(t)$. Approximation of the $O_2$ pressure in the bubble. To obtain the net flux across the diffusion barrier with $\varphi_i(t)$, we need to use $K(t)$. According to Fick’s first law (33), the flux of $O_2$ passing through the surface of the boundary layer at time $t$, $J_{O_2}$, is approximately given by

$$J_{O_2} = \frac{-D_{O_2}}{RT} \left[ s_{i,O_2} [-\Phi_{O_2}(t)] - P_{b,O_2}(t) \right] \tag{A9}$$

From the law of conservation of mass (14, 33), we have

$$A(t)J_{O_2}(t) = M_{b,O_2}(t), \quad t > t_b \tag{A10}$$

It has been shown that, for $N_2$, linear gas exchange kinetics in tissues are invoked for long half times for which the tension of dissolved gases exceeds ambient pressure (30, 31). With the assumption of similar kinetics for $O_2$, $M_{b,O_2}(t)$ is approximately linear in $t$ for $t > t_b$. Therefore, we readily obtain $M_{b,O_2}(t) = M_{b,O_2}(t_b)$. Also, according to Henry’s Law, we can write

$$M_{b,O_2}(t) = \frac{V_b(t)}{RT} \left[ s_{i,O_2}h\Phi(t) - P_{b,O_2}(t) \right] \tag{A11}$$

From Eqs. A9–A11, it follows that

$$P_{b,O_2}(t) = s_{i,O_2}\Phi(t) - 3D_{O_2} \frac{hR_b(t)\Delta t}{R_b(t)\Delta t + 3D_{O_2}\Delta t}, \quad \Delta t = t - t_b \tag{A12}$$

(Recall that $h = Vti/V_b$ is assumed constant.)

### Relationship between $O_2$ and $N_2$ tissue half times.

Let $Q(t)$ be the blood flow in the homogeneous tissue with volume $Vti(t)$ at time $t$. Also let $s_{i,N_2}$ and $s_{i,N_2}$ be the solubilities of $N_2$ in blood and tissue, respectively, and let $s_{i,O_2}$ and $s_{i,O_2}$ be similar characteristics for $O_2$ (Table 2). The half times $k_1$ and $k_2$ for $N_2$ and $O_2$ (4, 14) are then given by

$$k_1 = \frac{s_{i,B}Q(t)}{s_{i,N_2}Vti(t)} \tag{A13}$$

where $i = 1$ or 2. Therefore, given $k_2$, we may obtain $k_1 = \frac{\Omega k_1}{k_2}$ without knowledge of blood flow and tissue shell volume, where $\Omega$ is a constant $\Omega_{2/2,1}N_2 = (1/\Omega_{2/2,1/2,1}N_1 + 1/\Omega_{2/2,1/2,1}N_1) \approx 0.022730.02 \times (0.0150/0.146) \approx 0.87$. Generation of simulated bubble creation times. In terms of our application, Parzen’s result can be stated as follows: “Given $N_i(T)$, the bubble creation times $t_{b,ij}$ are distributed as order statistics corresponding to $N_i(T)$ independent random variables with common cumulative distribution function $F(t) = m(t/m(T)) (talt \leq t \leq T)$. In accordance with this result, we generated the unordered times $t_{b,ij}$ by $t_{b,ij} = F^{-1}(U_{ij}) = -(1/\beta\log[1 - U_{ij}(1 - e^{-\beta}))])$, where the $U_{ij}$ were independent uniform (0,1) random variates $[j = 1, \ldots, N_i(T)]$. The $t_{b,ij}$ were then sorted in ascending order to obtain the $t_{b,ij}$. The authors acknowledge Drs. Bruce D. Butler, Joseph R. Rodarte, and Michael B. Reid for critically reading the manuscript. The authors thank Dr. Johnny Conkin for useful advice and Dr. Michael L. Gernhardt for the many discussions. This study was supported by National Aeronautics and Space Administration Cooperative Agreement NCC9-58. 

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